





# *A Multiple Model Predictive Control for Maximum Energy Extraction from Variable Speed Wind Power Systems*

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**Abstract**— Due to concerns about the environment pollution caused by the burning of fossil fuels, and its continually diminishing reserves, the use of wind power for generating electricity has been increasing over last few decades. Nowadays, controller design for a variable speed wind turbine is one of the most challenges for engineers.

In this paper, a control strategy based on multiple model predictive control techniques for the control of variable speed wind turbines in the below rated wind speed regime is proposed. In this region control objectives are mainly to maximize energy capture, and to reduce dynamic loads. This has the effect of increasing the efficiency and the lifetime of the wind energy conversion system (WECS). Furthermore, in this control structure the constraints on the system variables in the controller design are considered and a multiple model structure to deal with the nonlinearities in the system is used.

A 2MW wind turbine is considered to show the good performances brought by the proposed approach by presenting and discussing the simulation results.

**Keywords**-variable speed wind turbine; maximum power point tracking; multiple model; predictive control

## I. INTRODUCTION

Wind energy has widely grown during the last decades. Nowadays, Control design for a variable speed wind turbine is one of most challenges to engineers. Implementation of advanced control systems is considered as a promising way to improve efficiency of power generation and to decrease wind energy cost. The wind turbine control objectives are mainly to optimize wind energy conversion, and to reduce dynamic loads. Indeed, dynamic loads affect wind turbines lifetime and consequently their costs.

Usually control systems designed for variable speed pitch regulated wind turbine consists of two loops to perform both increasing power output and keep wind turbine safety over the whole operation region. At below rated wind speed, the optimal tip speed ratio is traced with capturing more wind energy to maximize power output by adjusting the turbine rotational speed (partial load region). At above rated wind speed, in the full load region, the wind turbine is controlled to reduce loads by producing a rated power output at a constant

turbine speed, which is obtained by controlling the pitch angle of the turbine's blades. These days there exists an increasing interest in the control of variable speed wind turbine in the partial load regime and in this paper only operation in this region is considered [1].

Design of wind turbine controller is not a straightforward task. The nonlinear behavior of the system, the stochastic nature of the wind, the uncertainties in the parameters of the models and external disturbances make controller design task more difficult.

The control problem in this region can be split into two separate control levels.

At first control level, to extract maximum energy from wind, a maximum power point tracking (MPPT) control is necessary to adjust the turbine speed according to the variation of wind speed. Many MPPT algorithms can be used. One alternative is to adjust to change linearly with the effective wind speed such that the tip speed ratio is always kept at its optimal value[2]. Many MPPT strategies were proposed by making use of the wind turbine maximum power curve [3], but the knowledge of the turbine's characteristics is required. In comparison, the hill climbing searching (HCS) MPPT is popular due to its simplicity and independence of system characteristics [4] -[5].

The second level is used to control the turbine speed and generator power to follow their desired values. At this level, the usual implemented controllers are calculated from a linearization of the model around an operating point. The design of PI controller is described in [6]. Due to the stochastic operation conditions and uncertainties in the system, the linear control method hasn't ideal system performance. To cope with the system nonlinearity, local controllers are designed at different operating points, and gain scheduling techniques are used. A gain-scheduling LQG controller in [7]. The nonlinear control method such as the sliding mode controller is used in [8].

In this paper, a control strategy based on multiple model predictive control techniques for the control of variable speed wind turbines in the below rated wind speed region is proposed. In this control structure the constraints on the

system variables in the controller design are considered and a multiple model structure to deal with the nonlinearities in the system is used.

The remainder of this paper is organized as follows. Section II and III describe the nonlinear model and linearization of wind turbine respectively. Section IV describes the control problem in the partial load region. Section V introduces the multiple model predictive controller. Simulation results are shown in Section VI. Finally, Section VII concludes this paper.

## II. NONLINEAR MODEL OF WIND TURBINE

The structure of a variable speed, wind energy conversion system can be structured as several interconnected subsystem models as it is presented in Fig.1. Details of the individual blocks are given more [9].

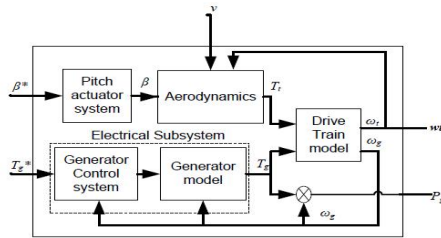


Figure 1. Wind turbine global scheme

### A. Aerodynamic System and Drive Train Model

A variable speed, horizontal axis wind turbine can be represented by a two mass model as shown in Fig. 2. Wind turbine power comes from the kinetic energy of the wind, thus it can be expressed as the kinetic power available in the stream of air multiplied by a  $C_p$  factor called power coefficient or Betz' factor. The power extracted by the wind turbine has the following expression:

$$p_{aero} = C_p(\lambda, \beta) \frac{1}{2} \rho \pi R^2 v^3 \quad (1)$$

Where  $\rho$  is the air density,  $R$  is the turbine radius, the power coefficient  $C_p$  is a nonlinear function of the blade pitch angle  $\beta$  and the tip speed ratio  $\lambda$  depending of the wind speed value  $v$  and the turbine rotational speed  $\omega_t$ , given by (2):

$$\lambda = \frac{\omega_t R}{v} \quad (2)$$

The aerodynamic torque,  $T_{aero}$  is the torque caught by the wind turbine, which is given by (3):

$$T_{aero} = \frac{c_p(\lambda, \beta)}{\lambda} \frac{1}{2} \rho \pi R^3 v^2 \quad (3)$$

Considering a flexible drive train model, the wind turbine can be described by the following differential :

$$\begin{aligned} J_T \frac{d\omega_t}{dt} &= T_{aero} - T_{mec} \\ J_{g-Ls} \frac{d\omega_{g-Ls}}{dt} &= T_{mec} - G_g T_g \\ \frac{dT_{mec}}{dt} &= k(\omega_t - \omega_{g-Ls}) + d \left( \frac{d\omega_t}{dt} - \frac{d\omega_{g-Ls}}{dt} \right) \end{aligned} \quad (4)$$

Where  $T_{mec}$  is the low-speed shaft torque,  $J_T$  and  $J_{g-Ls}$  are respectively the turbine and the generator (reported to the low-speed shaft) inertia,  $\omega_{g-Ls}$  is the generator rotational speed reported to the low speed shaft,  $G_g$  is the gearbox ratio,  $\omega_g$  is the generator rotational speed,  $k$  and  $d$  are respectively the mechanical coupling stiffness and damping coefficients,  $T_g$  is the generator torque [9].

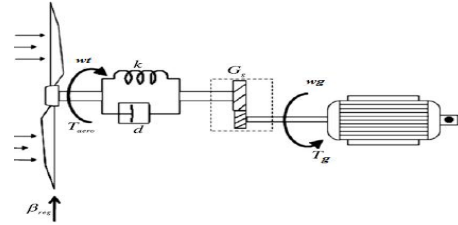


Figure 2. Wind turbine drive train dynamics

### B. Generator Model

For turbine controller design, it is important to use simple models that capture the relevant dynamics of the system. Fortunately, the dynamics of the electrical subsystem are much faster than the turbine dynamics and simple models can be used to represent the electrical dynamics. In this paper, a first order model, given in (5), (6) is used.

$$\frac{dT_g}{dt} = \frac{1}{\tau_g} T_g^* - \frac{1}{\tau_g} T_g \quad (5)$$

$$p_g = \eta T_g \omega_g \quad (6)$$

Here,  $P_g$ ,  $T_g$ ,  $\tau_g$  and  $\eta$  are the generator power, generator torque, time constant and efficiency, respectively [10].

## III. LINEARIZATION AND STATE SPACE REPRESENTATION

The nonlinearity of the system is due to the  $C_p$  characteristic, which is used in the expression of the aerodynamic torque. We need then to linearize the equation (3) of  $T_{aero}$  around an operating point defined by the wind speed value. Thereafter, the linearized state space model of the system can be written as (7):

$$\begin{aligned} \dot{x} &= Ax - Bu \\ y &= Cx + Du \end{aligned} \quad (7)$$

Where  $x$ ,  $y$  and  $u$  are respectively the state vector, control input and measured output defined as (8):

$$\begin{aligned} x &= [\omega_t \quad \omega_{g-Ls} \quad T_g \quad T_{mec}]^T \\ u &= [T_g] \\ y &= [\omega_t \quad p_g]^T \end{aligned} \quad (8)$$

And A, B, C and D are respectively the state, input, output and feed through matrices defined as follows:

$$A = \begin{bmatrix} \frac{a}{J_t} & 0 & 0 & \frac{-1}{J_t} \\ 0 & 0 & \frac{-G_g}{J_{g-Ls}} & \frac{1}{J_{g-Ls}} \\ 0 & 0 & \frac{-1}{\tau_g} & 0 \\ k + \frac{d \cdot a}{J_t} & -k & \frac{d \cdot G_g}{J_{g-Ls}} & -d \left( \frac{1}{J_t} + \frac{1}{J_{g-Ls}} \right) \end{bmatrix} \quad (9)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\tau_g} \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & G_g T_{g-op} & G_g \omega_{g-Ls-op} & 0 \end{bmatrix}$$

$$D = \text{Zeros}(2,1)$$

Where

$$a = \left( \frac{1}{2} \rho \pi R^3 \frac{v^3}{\omega_t} \left[ \frac{\partial C_p}{\partial \lambda} - \frac{C_p}{\lambda} \right] \right)_{op} \quad (10)$$

It can be seen that the system dynamics vary when the average wind speed varies.

#### IV. CONTROL PROBLEM DESCRIPTION

There are two main modes of operation of the variable speed wind turbine:

- (1) Partial load region:  $v_{cin} < v < v_{rated}$ .
- (2) Full load region:  $v_{rated} < v < v_{cout}$ .

Where  $v_{cin}$ ,  $v_{rated}$  and  $v_{cout}$  are cut-in, rated and cut-out wind speed respectively

The control system acts in a different way in each one. In the partial load region, the goal of the control system is to capture as much energy from the wind as possible while on the full load region its goal is to keep extracting the nominal power while avoiding overloads.

In this paper, only the partial load region is considered. The control problem in this region can be split into two separate control levels.

At first control level, a Maximum Power Point Tracking (MPPT) algorithm is used to calculate the turbine speed set point, so that the energy conversion efficiency is maximized. Many MPPT algorithms can be used. One alternative is to adjust to change linearly with the effective wind speed such that the tip speed ratio is always kept at its optimal value.

The second level is used to control the turbine speed and generator power to follow their desired values by manipulating the generator torque set and fixing the pitch

angle at its optimal value (usually very close to zero) [1], [10].

#### V. PROPOSED CONTROL STRATEGY

In this paper, a control strategy based on multiple model predictive control techniques for the control of variable speed variable wind turbines in the below rated wind speed zone is proposed.

##### A. Reviews of Model Predictive Control

Model-based predictive control (MPC) has been successfully used in many industrial applications in recent decades for many reasons such as: [11]

- Predictive control algorithms can take into account in a natural way constraints on both process inputs (control signals) and process output values (controlled variables), which often decide on the quality, effectiveness and safety of production.
- Predictive control can naturally be applied to multivariable process control, also when the numbers of the control inputs and the controlled variables differ.
- The principle of operation of these algorithms is comprehensible and relatively easy to explain to engineering, which is a very important aspect when introducing new techniques into industrial practice.

##### B. Predictive Control Principle

MPC is a digital controller, i.e. a discrete time technique. The general principle MPC can be understood from Fig. 3. Assume we are at certain sampling time  $k$ . The past trend for the output ( $y$ ) up to  $k$  and input ( $u$ ) up to  $k-1$  are known. The objective is then to find the future trend for the input that moves the future trend of the output approaches the desired reference trajectory  $r(k+1)$ . The control actions are found through iteration. In fact, an optimization problem is solved to compute online and in real-time the open loop sequence of present and future control moves  $[u(k|k), u(k+1|k) \dots u(k+N_c-1|k)]$ , such that the predicted outputs  $[y(k+1|k), y(k+2|k) \dots y(k+N_p|k)]$  follow the predefined trajectory. The optimization is solved taking into consideration constraints on the outputs and inputs. The first control action  $u(k|k)$  is then implemented on the real plant over the interval  $[k, k+1]$ . In the method,  $N_c$  is known as the control horizon and  $N_p$  as the prediction horizon.

At the next sampling time  $k+1$ , the prediction and control horizon are shifted ahead by one step and a new optimization problem is solved using updated measurements from the process. Thus, by repeatedly solve an open-loop optimization problem with every initial conditions updated at each time step, the model predictive control strategy results in a closed-loop constrained optimal control technique [12].

### C. Multi-Model Description

Many researchers advocated the use of the multi model approach in the modeling, analysis and control of nonlinear complex systems.

In this approach, the whole operating region is divided into M sub-regions with M linearized models that adequately represent the local system dynamics within each sub-region. A linear controller based on each model is designed. Finally, a criterion by which the control system switches one controller to another as operating conditions change is defined [13].

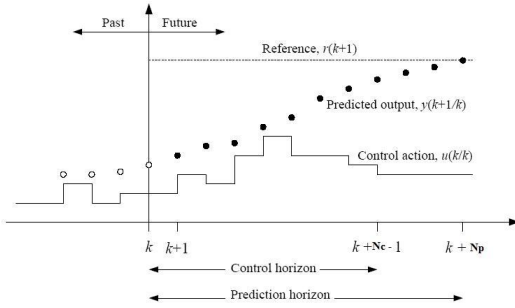


Figure 3. MPC principle

### D. Multi-Model Predictive Control

Controller designs for the proposed action are as follows:

- First, the wind turbine system is approximated with M linear models that built a hybrid state space model:

$$\begin{aligned} x^i(k+1) &= A^i x^i(k) + B_u^i u(k) + B_d^i d^i(k) \\ y^i(k) &= C^i x^i(k), i=1, \dots, M \end{aligned} \quad (11)$$

In equation (11), A(k), B(k) and C(k) are computed from equation (9) at the sampling instant k, and d(k) is used to represent the effect of actual unmeasured disturbance, it is modelled as the output of the system with Gaussian white noise  $n_d$  as the input:

$$\begin{aligned} x_d(k+1) &= \bar{A} x_d(k) + \bar{B} n_d(k) \\ d(k) &= \bar{C} x_d(k) + \bar{D} n_d(k) \end{aligned} \quad (12)$$

- Second, a linear MPC controller based on each model is designed. The optimization problem involving the physical constraints on the variables of the controlled system, such as limits on the generator torque, generator power and turbine speed. The optimization cost function is given by equation (13):

$$J(N_p, N_c) = \sum_{j=1}^{N_p} Q[y(k+j|k) - y^*(k+j)]^2 + \sum_{j=0}^{N_c-1} R[\Delta u(k+j)]^2 \quad (13)$$

$$0 \leq T_g^*(k+j) \leq T_{g-\max}$$

Subject to:  $\omega_t^i(k+j) \leq \omega_{t-\max}$

$$p_g^i(k+j) \leq p_{g-\max}$$

Where  $y^*$  is reference output,  $\Delta u(k)$  is defined as  $u(k)-u(k-1)$  and  $\{Q, R\}$  are weighting coefficient matrices.

- Finally, a criterion for switching between different controller is defined. In this paper, switching between different MPCs is based on the value of average wind speed.

This control scheme is presented in Fig. 4. [10], [14].

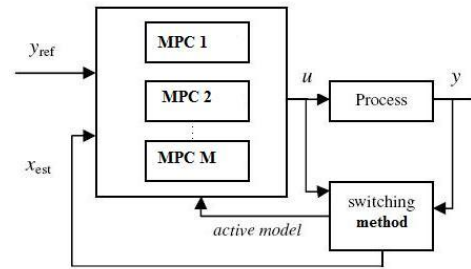


Figure 4. Multiple model predictive control scheme

## VI. SIMULATION RESULTS

The performance of the proposed control strategy is assessed in this section. In all simulations provided in this section, the designed controllers are tested on the nonlinear wind turbine model. The proposed control strategy has been implemented on a simulated 2 MW wind turbine. The parameters of the simulated plant and MMPC parameters such as the sampling time, the prediction horizon, the control horizon are given in Table I. [9]

First, the performance of the single model predictive control (SMPC) is compared with multiple model predictive control (MMPC) and Second, we compare Performance of the MMPC controller with the classical PID control strategy.

TABLE I: Wind turbine parameter value

Parameter	Value
Turbine radius	40m
Optimum value of $\lambda$ ( $\lambda_{opt}$ )	9
Maximum power coefficient	0.477
Nominal turbine speed	2.47 rad/s
Rated wind speed	11 m/s
Cut-in wind speed	5 m/s
MMPC parameters	$T_s=50ms, N_p=20, N_c=5$

### A. SMPC and MMPC

In this section, the performance of SMPC is compared with MMPC, The MMPC uses four linearized models (M=4). A simulation results are shown in Fig. 5- Fig.9.

If we calculate the standard deviation of the tip speed ratio from its optimal value and average power generated, we obtain that with SMPC, these are 0.61, 6.58\*100000 respectively and for MMPC, these are 0.58, 6.67\*100000. Results indicates a slight increase in the average power produced and decrease in the standard deviation of  $\lambda$  around its optimal value when using the MMPC controller.

In general, increasing the number of partitions will enhance the linear approximation and the prediction accuracy. This comes at the cost of increasing the controller complexity.

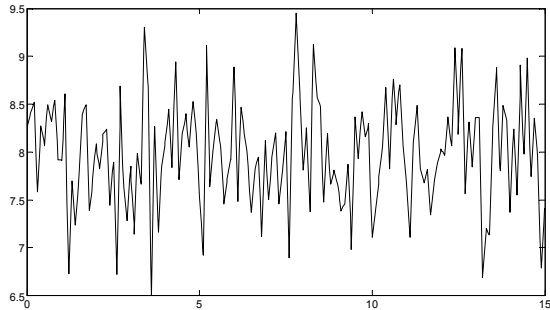


Figure 5. Wind speed

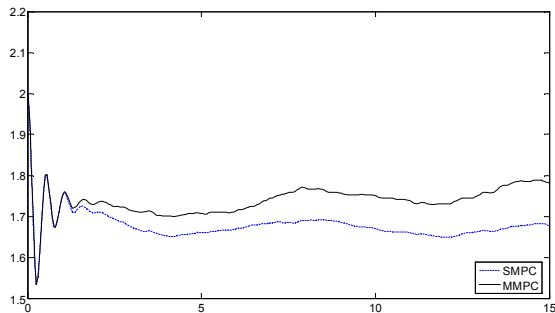


Figure 6. Turbine speed

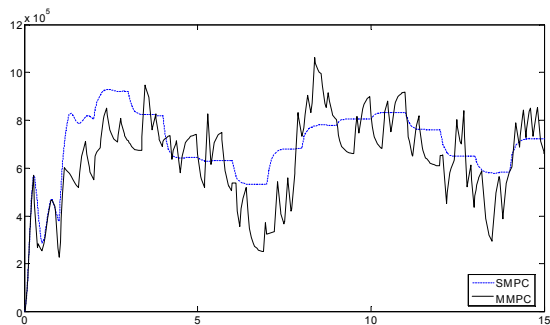


Figure 7. Generator power

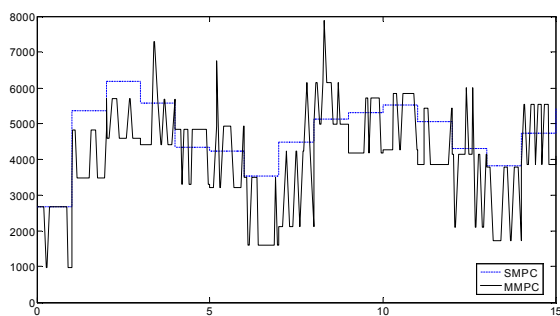


Figure 8. Generator torque

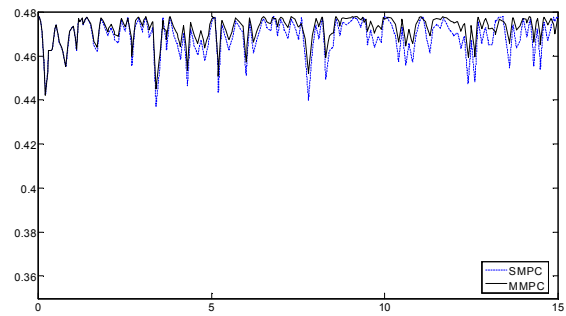


Figure 9. Power coefficient

### B. MMPC and PID

In this section, we compare Performance of the MMPC controller with the classical PID control strategy.

If we calculate the standard deviation of the tip speed ratio from its optimal value and average power generated, we obtain that with PID, these are 0.56 ,  $7.23 \times 100000$  respectively and for MMPC, these are 0.54,  $7.44 \times 100000$ . Results indicates a slight increase in the average power produced and decrease in the standard deviation of  $\lambda$  around its optimum value when using the MMPC controller. A simulation results are shown in Fig. 10- Fig. 14.

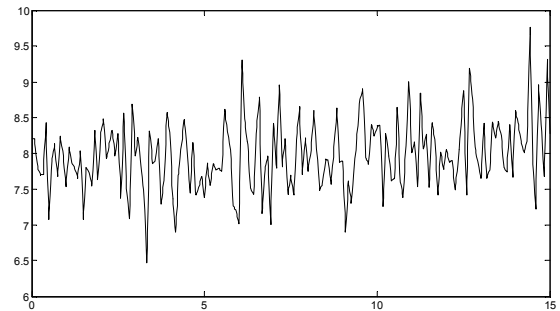


Figure 10. Wind speed

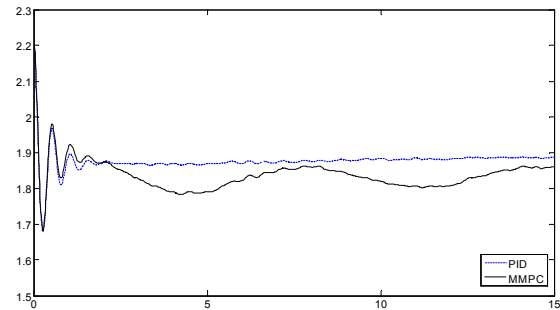


Figure 11. Turbine speed

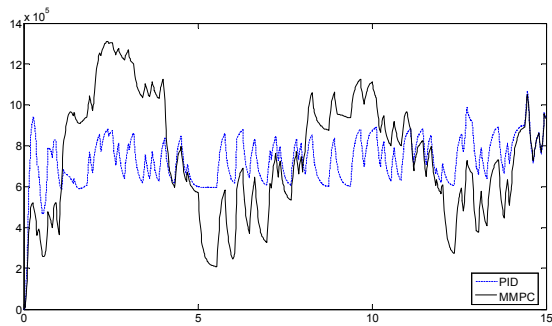


Figure 12 Generator power

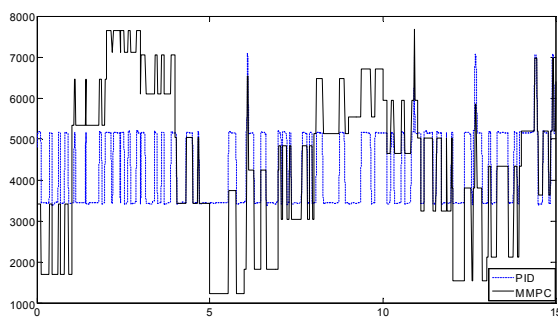


Figure 13. Generator torque

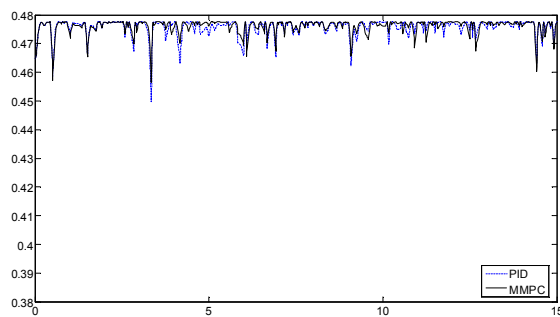


Figure 14. Power coefficient

## VII. CONCLUSION

In this paper, multiple model predictive control is proposed to control wind turbine in the partial load region. The advantages of this control structure are to consider the constraints on the system variables in the controller design and using a multiple model structure to deal with the nonlinearity in the system. Multiple model strategy causing good performance of the closed-loop system over the whole operating region in the partial load regime. The MMPC

controller can be designed to provide the desired trade-off between energy maximization and reduction of dynamic loads experienced by the plant mechanical structure. The performance of the MMPC controller is compared with SMPC and the traditional PID controller. Simulation results show that the MMPC controller provides much better power capture. This has the effect of increasing the efficiency and the lifetime of the wind energy conversion system.

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