SYSTEM IDENTIFICATION

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References are appeared in the last slide.

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Lecture 1

Perspective on System Identification

Topics to be covered include:

- Introduction.
- System Identification.
- Place System Identification on the global map. Who are our neighbors in this part of universe?
- The system identification procedure.
- Prerequisites.
- An Example of System Identification.
- References.

Some Definitions

System: An object in which variables of different kinds interact and produce observable signals.

Stimuli: External signals that affects system.

Static System: A system that the current output value depends only on the current external stimuli.

Dynamic System: A system that the current output value depends not only on the current external stimuli but also on their earlier value.

Time series: A dynamic system whose external stimuli are not observed.

Dynamic systems

Stimuli

Input

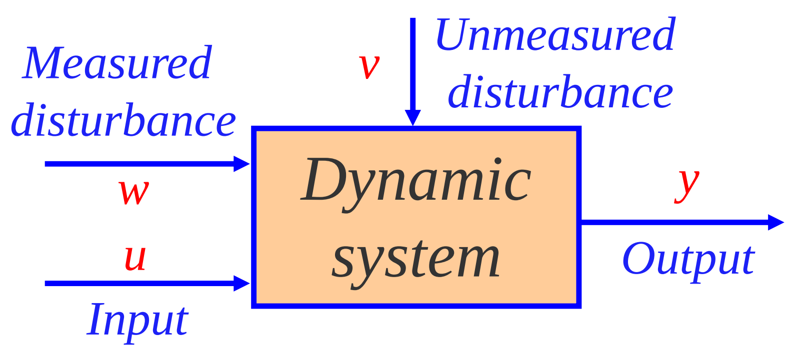
It can be manipulated by the observer.

Disturbance

It can not be manipulated by the observer.

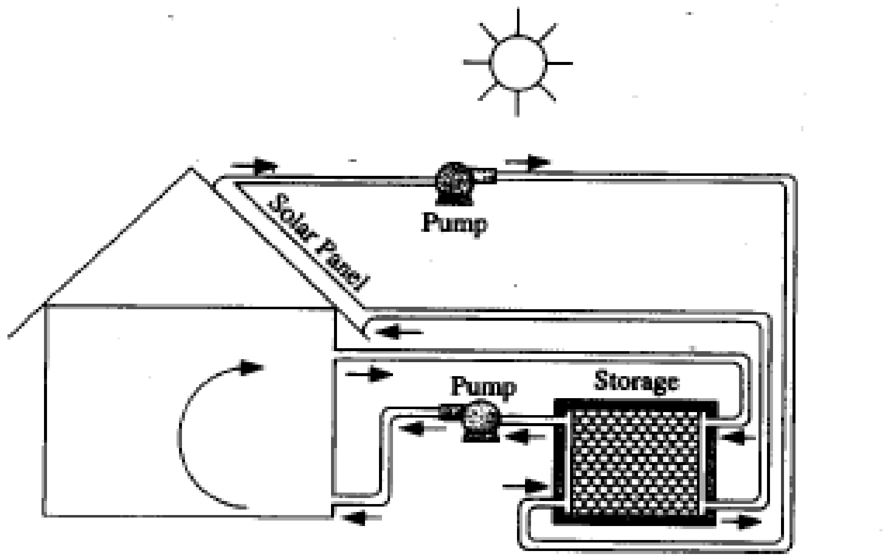
Measured

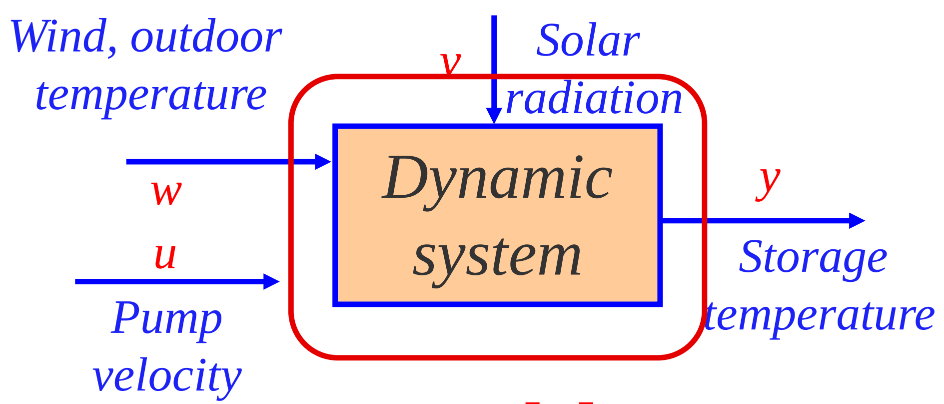
Unmeasured



Ali Karimpour Sep 2015

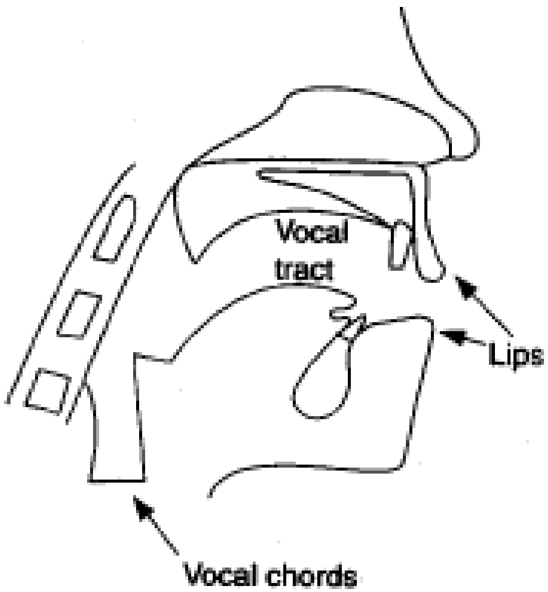
A solar heated house

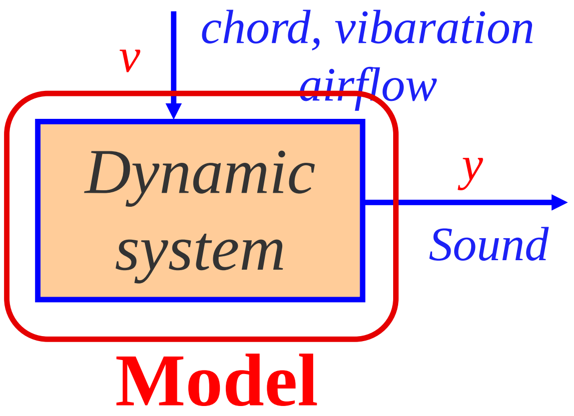




Model

Speech generation





Time series: A dynamic system whose external stimuli are not observed.

Models

Model:

A simplified representation of a limited part of reality with related elements.

A relevant system description from observed data.

Why we need model:

- Controller design.
- System simulation.
- Prediction.
- State predictor.
- Fault detection.

Models

Model: Relationship among observed signals.

Building models

1- **Modeling**
- Split up system into subsystems,
- Joined subsystems mathematically,
- Does not necessarily involve any experimentation on the actual system.

2- **System identification**
- It is directly based on experimentation.
- Input and output signals from the system are recorded.

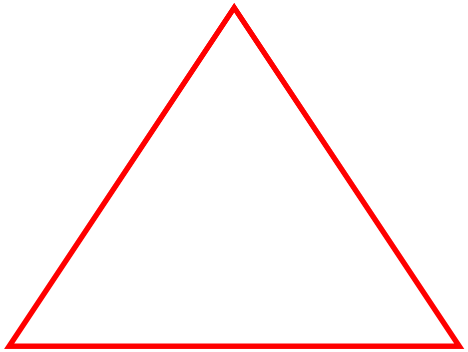
3- **Combined**

System Identification

System Identification: The art and science of building mathematical models of dynamic systems from observed input-output data.

System Identification is look for sustainable description by proper decision on:

Model complexity



Information contents in the data

Effective Validation

The fiction of a true model

The real-life actual system is an object of a different kind than our mathematical models.

Our acceptance of models should thus be guided by "usefulness" rather than "truth".

Nevertheless, we shall occasionally use a concept of "the true system"

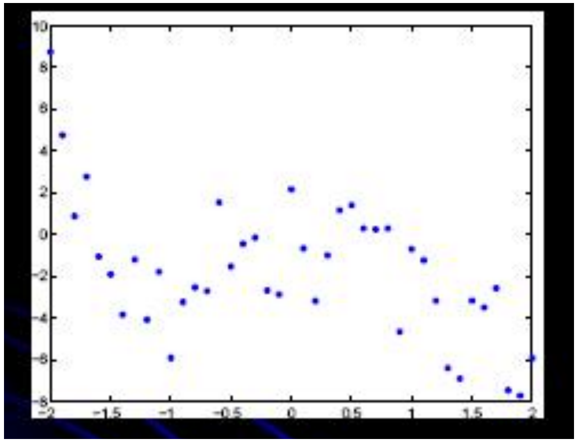
The Core

The Core: The core of estimating models is statistical theory.

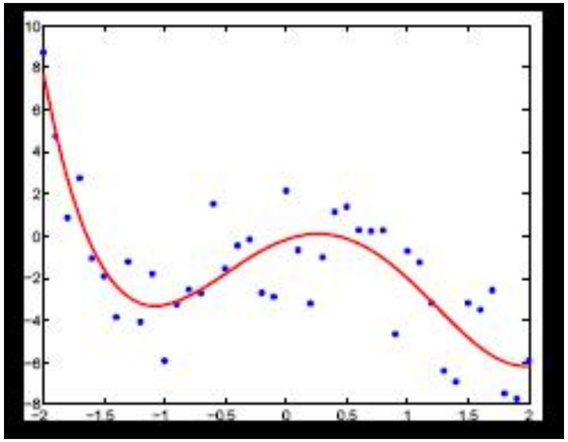
- Model: m
- True Description: S
- Model Class: M
- Complexity (Flexibility): C
- Information: Z
- Estimation
- Validation
- Model Fit: $F(m,Z)$

Estimation

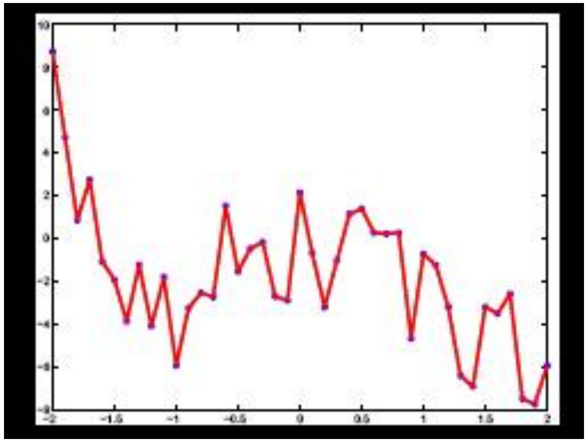
A template problem: Curve fitting



Squeeze out the relevant information in data.



No more satisfaction



All data contains signal and noise.

Estimation

The simplest explanation is usually the correct one. So the conceptual process for estimation is:

$$\hat{m} = \arg \min_{m \in M} \left[F(m, Z_e^N) + h(C(m), N) \right]$$

Fit measure

good agreement with data

Complexity measure

not too complex

$\hat{m}$ is a random variable since of irrelevant part of data (noise).

The System Identification Problem

1- Select an input signal to apply to the process.

2- Collect the corresponding output data.

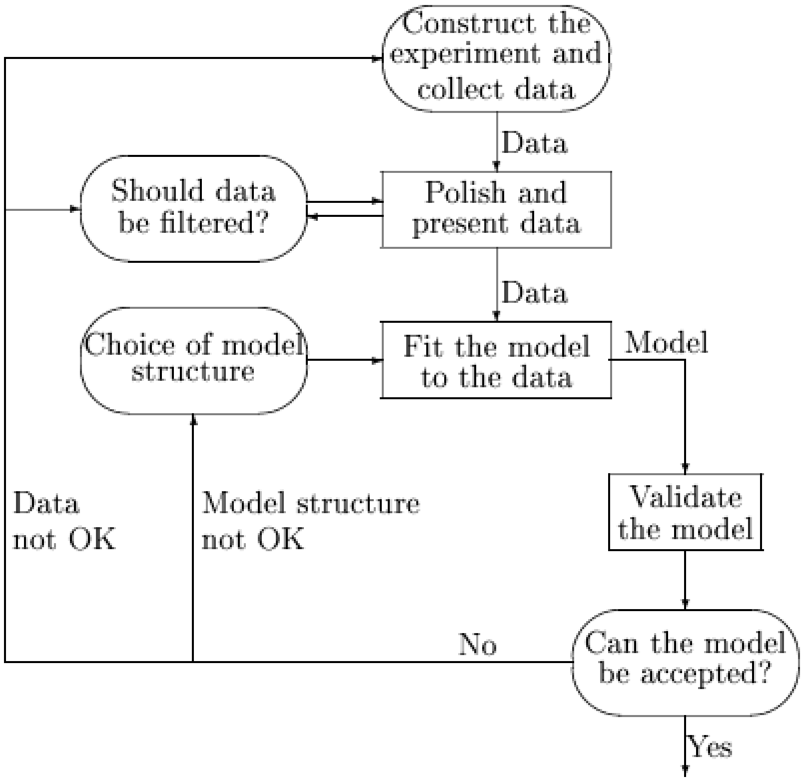
3- Scrutinize the corresponding output data to find out if some preprocessing …

4- Specify a model structure.

5- Find the best model in this structure.

6- Evaluate the property of model.

7- If the model is not adequate, go to step 4 or 3 or 1.



The System Identification Problem

1- Choice of Input Signals.

- Filtered Gaussian White Noise.
- Pseudo Random Binary Noise, PRBS.
- Chirp Signals or Swept Sinusoids.
- Random Binary Noise.
- Multi-Sines.
- Periodic Inputs.

2- Preprocessing Data.

- Drifts and Detrending.
- Prefiltering.

3- Selecting Model Structures.

- Looking at the Data.
- Examining the Difficulties.
- Accepting the Models .
- Getting a Feel for the Difficulties.
- Fine Tuning Orders and Noise Structures .

Model Structures

Model types

- **Parametric Models.**
  - **Linear Models.**
  - **Non-Linear Models.**
- **Non-Parametric Models.**
  - **Linear Models.**
  - **Non-Linear Models.**

Model types

- **Static Models.**
  - **Linear Models.**
  - **Non-Linear Models.**
- **Dynamic Models.**
  - **Linear Models.**
  - **Non-Linear Models.**

ARX as a Parametric Linear Models

Input-Output relationship.

AR part (Auto Regressive)

$$y(t)+a_1y(t-1)+\ldots+a_{n_a}y(t-n_a)=$$
$$b_1u(t-1)+\ldots+b_{n_b}u(t-n_b)+e(t)$$

X part (eXogenous)

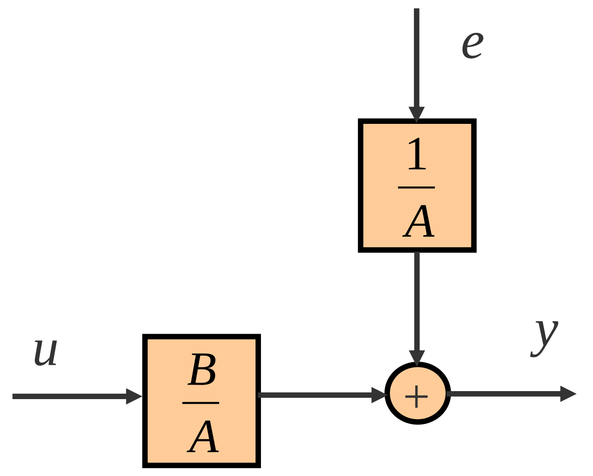
Adjustable parameters in this case are

$$\theta=\left[a_1\ a_2\ldots a_{n_a}\ \ b_1\ b_2\ldots b_{n_b}\right]^T$$

ARX model

Define

$$A(q)=1+a_1q^{-1}+\ldots+a_{n_a}q^{-n_a}$$
$$B(q)=b_1q^{-1}+\ldots+b_{n_b}q^{-n_b}$$



The ARX model structure

The Communities around the core

System Identification.

1- Statistics. ML Methods, Bootstrap method,…

2- Econometrics and time series analysis.

3- Statistical learning theory.

4- Machine learning.

5- Manifold learning.

6- Chemo metrics.

7- Data Mining.

8- Artificial Neural Network.

9- Fitting Ordinary Differential Equation to Data.

Prerequisites

1- Linear algebra.

2- Statistics fundamentals.

3- Stochastic Process.

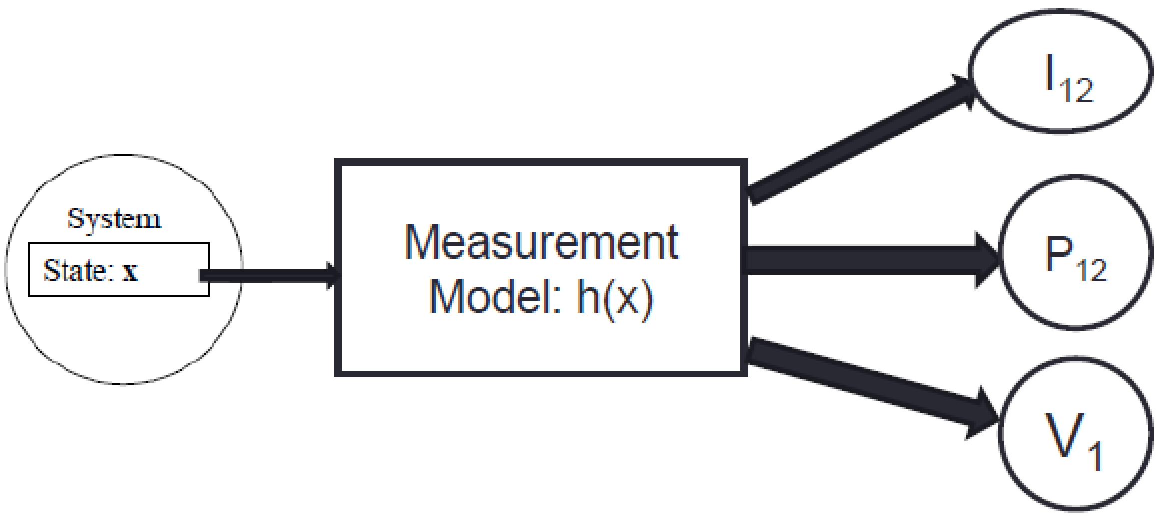
4- Matlab Software.

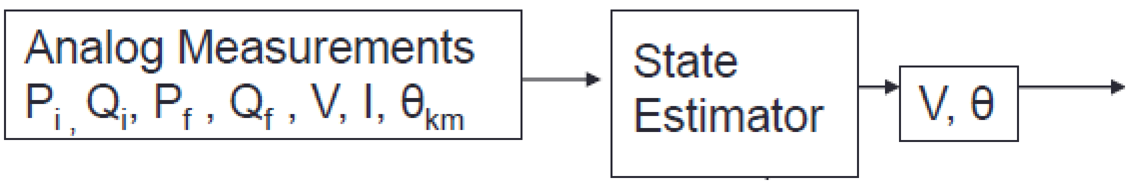
An examples in the field of system Identification

Power System State Estimation

The state (x) is defined as the voltage magnitude and angle at each bus.

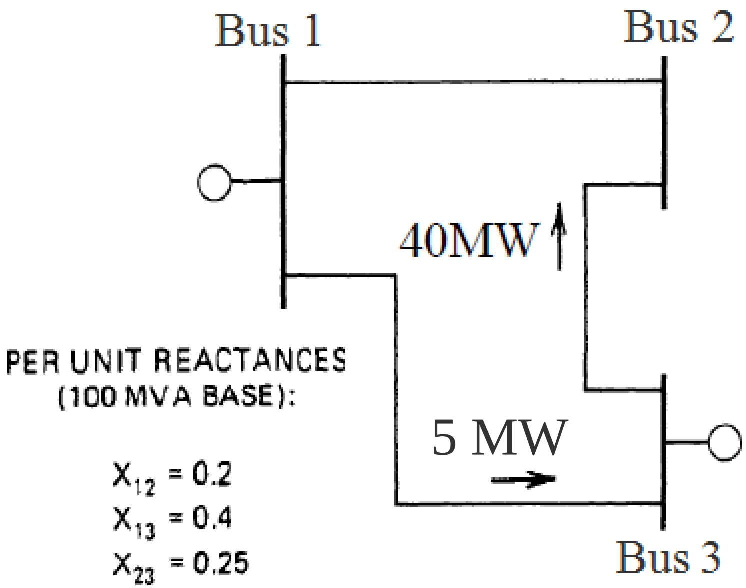
All variables of interest can be calculated from the state and the measurement mode.





An examples in the field of system Identification

Power System State Estimation



Let $\theta_3 = 0$ rad

$$P_{32} = \frac{|V_3||V_2|}{X_{23}} \sin(\theta_3 - \theta_2)$$

$$0.4 = \frac{1}{0.25}(\theta_3 - \theta_2)$$

....

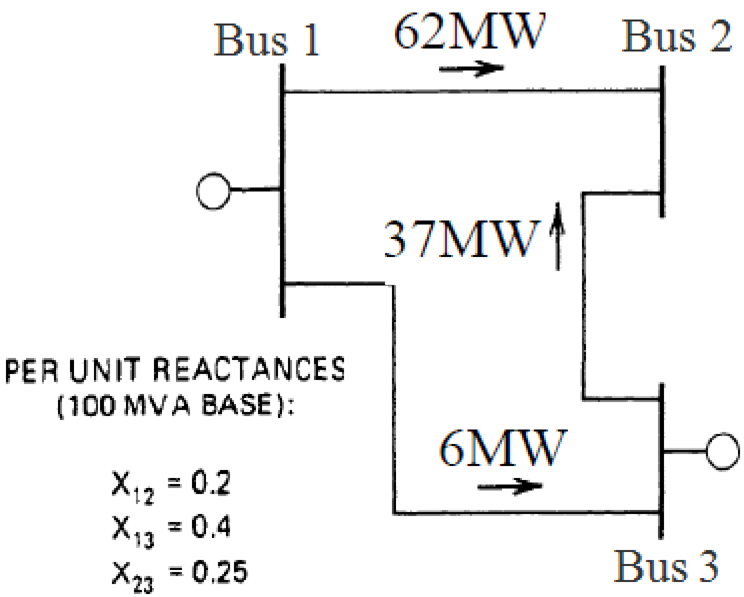
$$\theta_1 = 0.02\text{rad}$$

$$\theta_2 = -0.10\text{rad}$$

$$\theta_3 = 0\text{rad}$$

An examples in the field of system Identification

Power System State Estimation



1- Noisy measurement.

$$\theta_1 = 0.024\text{rad}$$
$$\theta_2 = -0.0925\text{rad}$$
$$\theta_3 = 0\text{rad}$$

$$P_{12} = 58.25\,\text{MW}$$

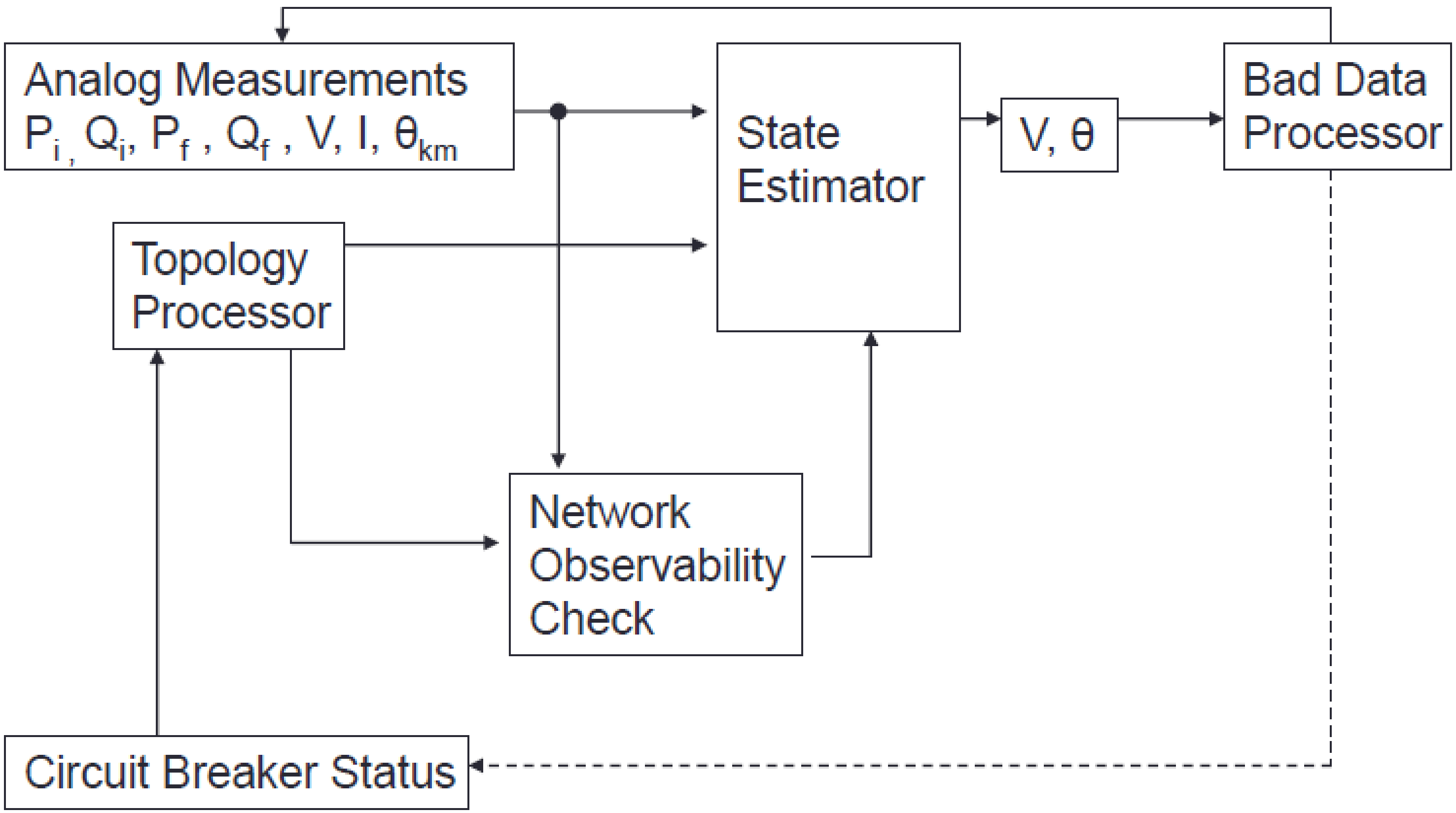
2- More Measurement.

Better estimation method ?

Bad data processor ?

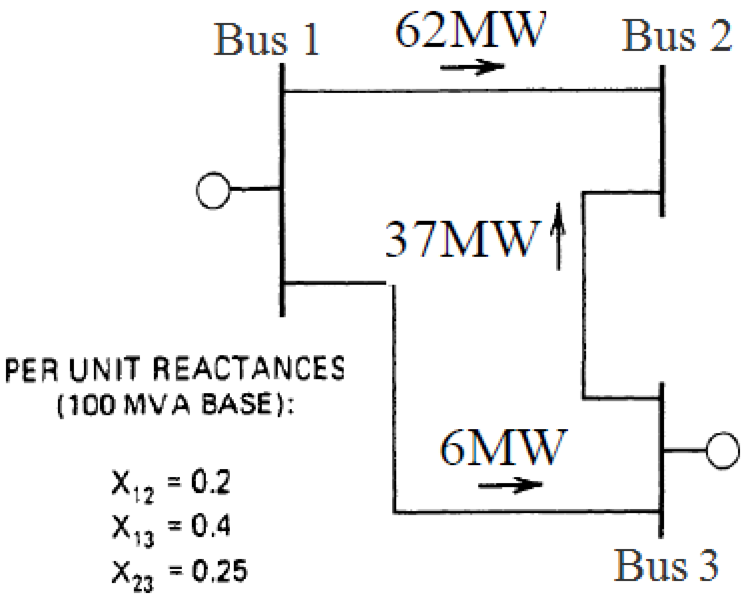
An examples in the field of system Identification

Power System State Estimation



An examples in the field of system Identification

Power System State Estimation



Better estimation method ?

$$P_{12} = \frac{1}{0.2}(\theta_1 - \theta_2) \qquad P_{12} = 5\theta_1 - 5\theta_2$$

$$P_{32} = \frac{1}{0.25}(0 - \theta_2) \qquad P_{32} = -4\theta_2$$

$$P_{13} = \frac{1}{0.4}(\theta_1 - 0) \qquad P_{13} = 2.5\theta_1$$

$$\begin{bmatrix} P_{12} \\ P_{32} \\ P_{13} \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 0 & -4 \\ 2.5 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\boxed{P = \varphi\theta}$$

Linear regression

• • •

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0.0286 \\ -0.0943 \end{bmatrix}$$

An Example of System Identification method

Perhaps the most basic relationship between the input and output is the linear difference equation

$$y(t)+a_1 y(t-1)+\ldots+a_n y(t-n)=b_1 u(t-1)+\ldots+b_m u(t-m)$$

$$y(t)=-a_1 y(t-1)+\ldots-a_n y(t-n)+b_1 u(t-1)+\ldots+b_m u(t-m)$$

$$y(t)=\varphi^T(t)\theta$$

Linear regression

Where

$$\theta=[a_1 \;\; \ldots.. \;\; a_n \;\; b_1 \;\; \ldots.. \;\; b_m]^T$$

$$\varphi(t)=[-y(t-1) \;\; \ldots.. \;\; -y(t-n) \;\; u(t-1) \;\; \ldots.. \;\; +u(t-m)]^T$$

$$\hat{y}(t|\theta)=\varphi^T(t)\theta$$

Least Square Method

Suppose that we don't know the value of parameters θ, but the recorded input and output over a time interval $1 \le t \le N$ is:

$$Z^N = \{u(1), y(1), \dots.. u(N), y(N)\}$$

Now define:

$$V_N(\theta, Z^N) = \frac{1}{N}\sum_{t=1}^{N}\left(y(t) - \hat{y}(t\,|\,\theta)\right)^2 = \frac{1}{N}\sum_{t=1}^{N}\left(y(t) - \varphi^T(t)\theta\right)^2$$

$$\hat{\theta}_N = \arg\min_{\theta} V_N(\theta, Z^N)$$

$$0 = \frac{d}{d\theta}V_N(\theta, Z^N) = \frac{2}{N}\sum_{t=1}^{N}\varphi(t)\left(y(t) - \varphi^T(t)\theta\right)$$

$$\sum_{t=1}^{N}\varphi(t)y(t) = \sum_{t=1}^{N}\varphi(t)\varphi^T(t)\,\theta$$

$$\hat{\theta}_N = \left[\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right]^{-1}\sum_{t=1}^{N}\varphi(t)y(t)$$

First order difference equation

Consider the simple model

$$y(t)+ay(t-1)=bu(t-1)$$

Then

$$\hat{\theta}_N=\left[\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right]^{-1}\sum_{t=1}^{N}\varphi(t)y(t)$$

Now we have

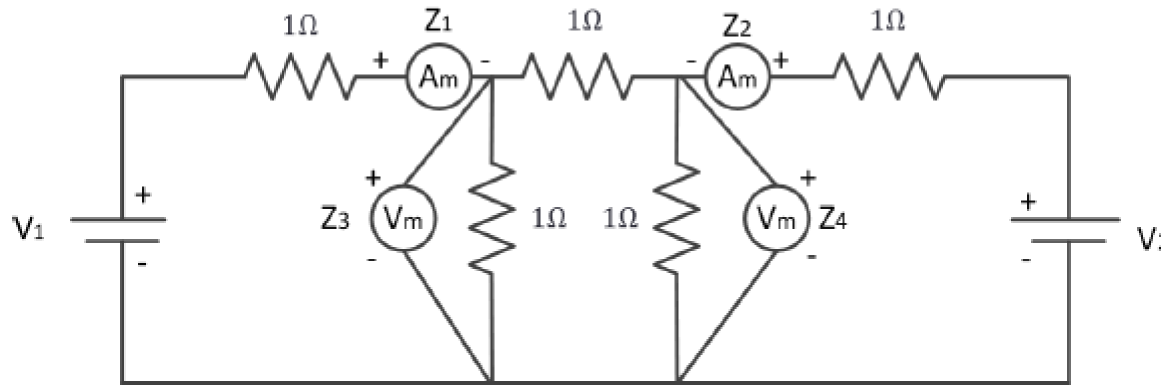
$$\varphi(t)=\begin{bmatrix}-y(t-1)\\ u(t-1)\end{bmatrix}\qquad \varphi(t)^T=[-y(t-1)\ \ u(t-1)]$$

$$\varphi(t)\varphi(t)^T=\begin{bmatrix}y^2(t-1) & -y(t-1)u(t-1)\\ -y(t-1)u(t-1) & u^2(t-1)\end{bmatrix}\qquad \varphi(t)y(t)=\begin{bmatrix}-y(t)y(t-1)\\ y(t)u(t-1)\end{bmatrix}$$

$$\hat{\theta}_N=\begin{bmatrix}\sum y^2(t-1) & -\sum y(t-1)u(t-1)\\ -\sum y(t-1)u(t-1) & \sum u^2(t-1)\end{bmatrix}^{-1}\begin{bmatrix}-\sum y(t)y(t-1)\\ \sum y(t)u(t-1)\end{bmatrix}$$

Exercises

Exercise1-1: Consider following system:



Consider the meter reading as Z_1=9.01 A, Z_2=3.02 A, Z_3=6.98 V and Z_4=5.01 V.

a) Derive Z_1, Z_2, Z_3 and Z_4 according to V_1 and V_2.

b) Derive V_1 and V_2.

c) Derive Z_1, Z_2, Z_3 and Z_4 from part “a” and compare with measured value.

d) Discuss the situation that a measurement is too noisy.

Exercises

Exercise1-2: Suppose the value of input and output of a system for t=1 to 6 u are:

$$u = \begin{bmatrix} 2 & 3 & 2 & 5 & 3 & 7 \end{bmatrix}^T \qquad y = \begin{bmatrix} -23 & 48 & -93 & 188 & -371 & 745 \end{bmatrix}^T$$

a) Consider the model of system as follows and derive model parameters ($\hat{\theta}_N$).

$$y(t) + ay(t-1) = bu(t-1)$$

b) Check the validity of model.

c) Consider the model of system as follows and derive model parameters ($\hat{\theta}_N$). Discuss the results!

$$y(t) + a_1 y(t-1) + a_2 y(t-2) = b_1 u(t-1) + b_2 u(t-2)$$

Exercises

Exercise1-3: Consider following system.

$$y(t)+ay(t-1)=bu(t-1)+e(t)$$

Suppose the value of input of the system for t=1 to 14 is:

$$u=\begin{bmatrix}2 & 3 & 2 & 5 & 3 & 7 & 0 & 2 & -1 & -5 & 5 & 3 & 2 & 3\end{bmatrix}^T$$

Let e(t) as a noise.

a) Consider a and b by yourself and let e(t) as a noise (between -1 and 1) derive y(t).

b) Derive model parameters ($\hat{\theta}_N$).

c) Check the validity of the model.

d) Repeat parts a, b and c and discuss(Same parameters and different noise and noise level).

References

1- "System Modeling and Identification" Rolf Johansson (2010)

2- "System Identification Theory For The User" Lennart Ljung(1999)

3- "Perspectives on System Identification" Lennart Ljung (2009)

4- "Lessons in Digital Estimation Theory" Jerry M. Mendel (1987)

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6- "System Identification" M. Karrari (2012) (Farsi)